# OPTIMIZATION OF THE ROTOR-WING SYSTEM FROM HELICOPTER PERFORMANCE POINT OF VIEW

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Summary - The subject dealt with by this paper is the optimization of geometric parameters of the helicopter lifting system in the form of the rotor and wing units.

Considering the numerous couplings of the parameters of the aforesaid lifting system with the remaining parameters of the helicopter, it may be said that the problem lies in the optimization of the entire helicopter, equipped with an auxiliary wing, and with particular attention paid to the rotor and to the wing.

Efforts have been undertaken to obtain a multi-objective estimate of the quality of the helicopter, while recognizing a multi-parameter set of design variables.

Considering the significant volume of the problem, the proceedings were limited to an outline of the course of optimal projecting, while illustrating the selected fragments of the process.

## 1. INTRODUCTION

Technical progress in the field of helicopter designing, and especially the tendency to achieve higher air speeds, resulted in developing new types of helicopters, provided with an auxiliary wing.

A wing of that kind, unloading the rotor in the level flight, displaces the stall limit on the rotor blades in the direct of higher air speeds. The wing also has a disadvantageous effect, mainly on the performance of hovering flight,due to the increase of helicopter weight, and the aerodynamic load caused by the flow behind the rotor, and for articulated rotors, deteriorates the handling qualities in such conditions of flight as tend to cause increased unload of the lifting rotor.

An overall estimate of the rotor-wing system permits a correct selection of parameters of the lifting system in order to obtain the required performance of the helicopter.

Utilization of computers permits to introduce optimization technique in designing helicopters an a larger scale. It was possible, therefore, to undertake work /since 1969/ tending to organize an effective auxiliary design tool for teams employed in projecting helicopters.

It was imperative, however, to work out such a system which would permit a multiobjective evaluation of the helicopter, considering that, until recently, the designers were obliged to come to conclusions of multiobjective character in an arbitrary way, and basing on their own experience and, perhaps, intuition as well.

Most often, this system allows to meet the requirements of regulations for helicopter construction but to ensure the competetivity of the aircraft, it is necessary to maximilize the quality of the helicopter. For working out an appropriate optimization method by means of computer calculations the principle was adopted to simulate the designing process; the area of admissible solutions was gradually reduced as the design process evolves. The optimization process was divided into two stages.

The first stage pertains to the conception of a project. This is mainly concerned with an overall estimate of the helicopter design, with an analytical penetration of a wide field of operationability, with an analysis of utilization of the particular configurations of the helicopter, and the review of the development possibilities of the project. Owing to a rather insignificant number of informations about a helicopter at this stage, the optimization concerns the essential parameters of the design. The mathematical model of the helicopter may thus include certain simplifications. This permits to increase the speed of calculating while maintaining an adequate level of evaluation of the project.

The second stage is concerned with the technical project. In this phase the indispensable set of the design parameters are covered by the optimization process in such flight conditions as were considered in the previous stage as critical items of the helicopter design. Considerable precision as well as accurate verification of the mathematical model of the helicopter are required at this stage.

The problem of connecting the automation of the optimization [1] with the multiple aspects of the evaluation of the project [2] constitutes the main difficulty of the art even if computers of the highest generation are employed. The break-down of the designing process into stages, the recognition of the hierarchy of parameter importance, the widely assumed iteration of the proceedings may tend to facilitate the solving of the problem. With this feature in mind, the following chapters of this paper will be presented.

## 2. THE OPTIMIZATION OF THE PROJECT CONCEPTION

The actual conceiving of a helicopter design, which would ensure the possibility of developing the construction as well as further modifications thereof, is the most essential stage of projecting. An erroneous conception detected at a later stage of the project realization, cannot be rectified by a modification of the values of design parameters of the helicopter, and more likely, an abandonment of the project is to be reckoned with.

At this stage, therefore, the many-sided aspects of the estimate of the helicopter are a foremost issue.

The accuracy of the mathematical model of the helicopter is of a somewhat lesser importance since in this case the optimization pertains to a relative estimate of the design variants, and the algorithm for analysis of the performance in the range of estimate of increments indicates lesser errors than when evaluating absolute quantities, by an order of magnitude.

The basis of the analytical program is the mathematical model of the helicopter, which includes the input data, i.e. the design parameters of the helicopter, and joins them to the output as the required flight properties. This model contains the procedure of representation of the steady forward flight; the procedure also includes the equations of motion in space /fig. 1/, and the estimate of the energy balance, necessary to calculate the basic performance, as well as the model of the helicopter dynamics for estimating the flying qualities /fig. 2/.

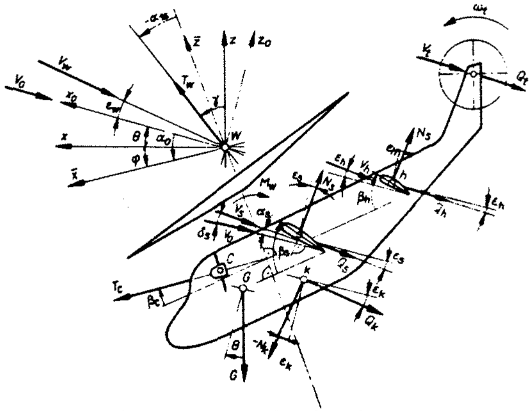

Fig. 1. Equilibrium of the helicopter in an oblique flight

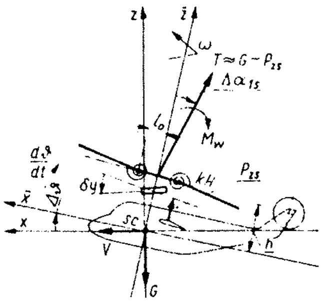

Fig. 2. Model of the helicopter dynamics on pitching direction

Criteria and formulae acc. to [3] were applied to estimate the directional manoeuvring. Magnitudes $\varepsilon_i$ designate the angles of downwash in the vicinity of the element "i" around which the flow takes place, and $N_i$ and $Q_i$ indicate respectively the aerodynamic lift, and the drag of the helicopter element.

As an example, in order to estimate the dynamics of the helicopter on the pitch direction, the motion analysis can be carried out by solving the characteristic equation of the configuration of equations of forces, and of moments binding with each other two degrees of freedom /displacement along axis $x$ and angle deflection $\vartheta$ /.

Equation of forces along axis x

$$G\,/g\ddot{x} - T\,/\vartheta + a_{1s}/ = 0$$

Equation of moments

$$-I\ddot{\vartheta}+Tha_{1s}+0.5k\,k_H a_{1s}+0.5k\omega^2 S_{pp} l_o a_{1s}=0$$

where:

$T$ - rotor thrust,

$a_{1s}$ - deflection of the blade tip plane from the plane of revolutions,

$I$ - inertia moment of the fuselage,

$k_H$ - elasticity of blade fastening in the flapping hinge /equivalent rigidity of a hingeless rotor/

The solution of the characteristic equation of the fourth order will permit to estimate the motion of the helicopter, and to calculate the time of oscillate T and the time of doubling the amplitude $T_2$ - in case of unstable motion /most frequently encountered condition in helicopter operation/.

The coefficients of moment equation permit to evaluate damping and sensitivity of control, required to estimate the flying qualities by means of Cooper's scale.

Mean quantities of loads and of induced velocities are employed in this mathematical model of a helicopter. Multiplication factors of the induced velocity in the environment of the lifting system of the helicopter, are assumed either by way of experiment or from a model of the vortex system.

Formulae employed in the procedure of analysing the properties of a helicopter, are based mainly on the momentum theory of the rotor and wing, and permit to estimate quickly the helicopter performance; such formulae embrace all the possible accuracies /a.o. by introducing semi-empirical correction factors/.

In the process of optimization it is possible to carry out an estimate of the construction with a fixed total weight of the helicopter, and including modifications of the weight of the construction /in case of altered values of the design parameters of the helicopter/ into the weight of the load or with a fixed weight of the load, assuming variation of the total weight.

In the second event it is necessary to estimate the increment of weight of the /n+1/-th variant of the helicopter $\Delta G = = \sum_i \Delta G_i$ with regard to the actually computed n-th variant of the helicopter with a G weight. $\Delta G_i$ is the increment of weight of a helicopter element, effected by the alteration of one of its design parameters by a value of $\Delta p_i$.

A simple method for estimating weight variations was assumed, e.g. for a rotor blade

$$\Delta G_b=G_b\underline{/}\,/1+\frac{\Delta b_{o7}}{b_{o7}}/\;/1+\frac{\Delta k}{k}/\;/1+\frac{\Delta R}{R}/^{2}-1\_7$$

where G - weight of blade of n-th variant, similarly to the blade chord $b_{o7}$ on radius 0.7R, number of blades k, and radius R.

The respective increments of parameters are $\Delta b_{o7}, \Delta k, \Delta R$. This method, although approximate, is sufficiently accurate for small increments $\Delta p_i$. It permits to control the optimization process with a recognition of not only the direct effect on the helicopter properties $W_j$, a modification of the parameters of helicopter elements, but also on the alteration of the helicopter weight $\Delta W_j=f\underline{/}\sum_i /\Delta p_i, \Delta G_i/\_7$, which is coupled to those parameters. This method is particularly advantageous for a comparative estimate of the helicopter configurations, for example a pure helicopter, and equipped with an auxiliary wing, especially for estimating the properties of hovering flight. The procedure of alterations of the helicopter weight caused by the alterations of its design elements, was programmed in a block, designated as "$\triangle$G", in diagram 3.

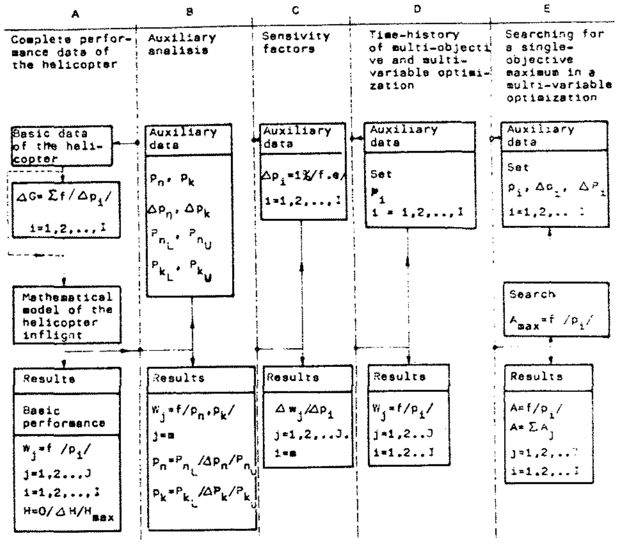

Fig. 3. Diagram of the optimization process at the stage of the conception project

The above mentioned basic cycle of calculations, i.e. the calculation of the set of helicopter properties for the set of its design parameter data, may be repeated in many a manner. Five of such consecutive sequences, illustrated in fig. 3, constitute an accepted pattern of procedure in the first stage of designing a helicopter. The iteration of optimisation proceedings requires a frequent repetition of the particular columns, not always in the sequence indicated in fig. 3; the essential course, however, consists in performing calculations from A to E.

Basing on knowledge, experience, statistical data, and on one's own intuition, it is necessary to establish the data of the initial variant of the helicopter, the so-called variant "0". Column A, repeating the basic cycle of calculations for different altitudes and climates, gives as a result the chart of basic performance and flying qualities of the helicopter /fig.4/. Optionally, in accordance with the requirements, a more or less voluminous assortment

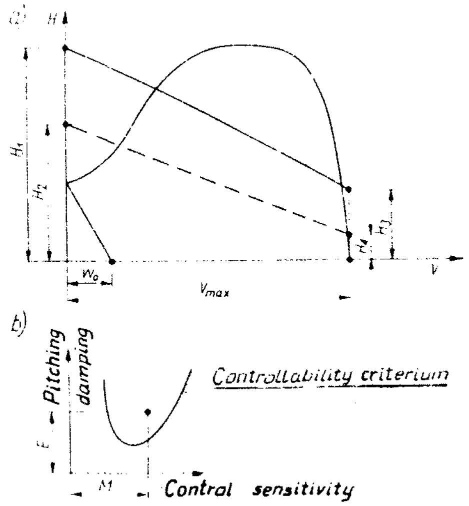

Fig. 4. Outline of the basic qualities of the helicopter estimated in the calculations program

of the helicopter qualities may be printed. Generally, it is possible to detect immediately the more rough errors with regard to the assumptions of the design parameters, or else the characteristic limitations of the operational range, to which attention should be turned at later stages of the analysis.

The next column B serves to exploration the environment of the properties, and the set of parameters of variant "0". By an alteration in the limits of the design variations of two selected parameters of the helicopter $p_k$ and $p_n$, it is possible to obtain a dependence of an arbitrary quality $W_j$ from those parameters $W_j = f/p_k, p_n/$, as the result of calculations in the form of a graph, with a recognition of the limits of their physical variation /for example the stall limit or compressibility/.

Those diagrams are in some measure a cross-section of the area of the bivariant dependence of the helicopter variables on the objectiv function. The generally target of this analysis is the orientation with reference to the character of the phenomena, to the possibility of occurence of an optimum of the objective function resulting both from the character of physical dependences and from the limitations. As an example, an illustration of variations of the necessary power $P_n$ is presented in hovering flight and in level flight for V = 250 km/h treated in this case as a objectiv in the function of radius R and velocity of the blade tip ωR, for a pure helicopter and equipped with an auxiliary wing /fig. 5, 6 and 7/. Another example is the deduction of power absorbed by the auxiliary wing in the function of its surface and air speed for the preset relief of the rotor /fig. 8/. On ground of those analyses the following conclusions can be drawn:

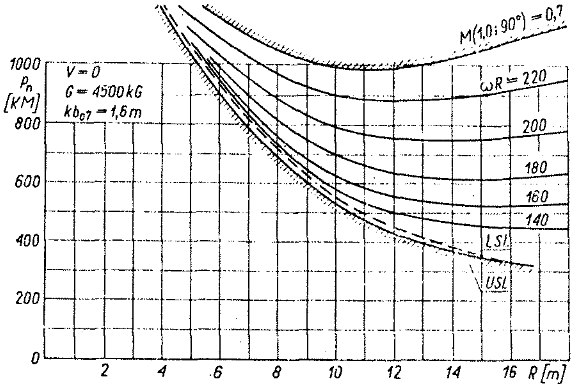

Fig.5. Dependence of necessary power $P_n$/R,ωR/ for the condition of hovering flight, and for different configurations of the helicopter

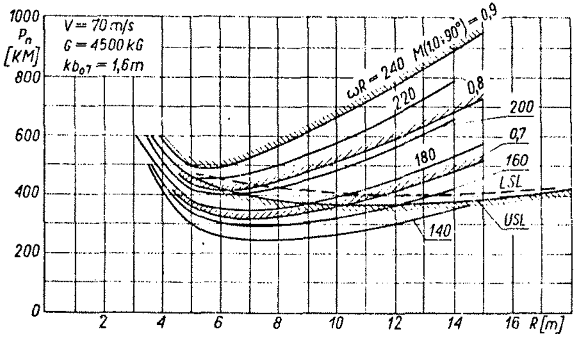

Fig.6. Dependence $P_n$ /R,ωR/ for a level flight at the air speed V=250 km/h for a classic configuration

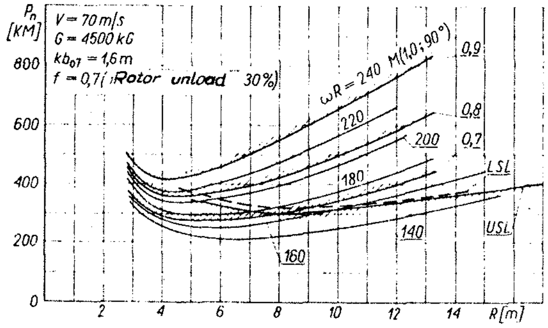

Fig.7. Dependence $P_n$ /R,ωR/ for a level flight at the air speed V=250 km/h for a configuration with an auxiliary wing

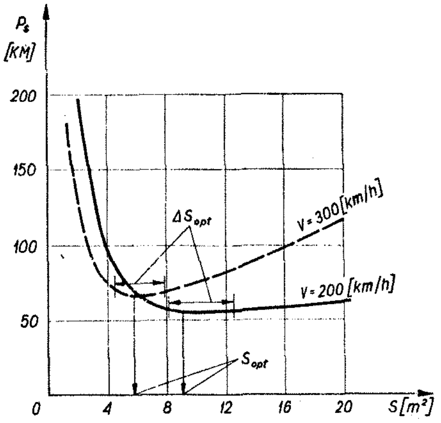

Fig.8. Dependence of power absorbed by the auxiliary wing of the helicopter in S surface function, and air speed V of the helicopter

- addition of the wing extends the admissible range of parameter selection with regard to the rotor /fig. 5, 6 and 7/
- the magnitude of the radius optimum, when flying at a high air speed is determined by the limiting curve USL /upper stall limit/ or by the LSL /lower stall limit/
- the flatness of the minimums enables a rather significant freedom of parameter selection in the present limits of variation /fig. 5, 6, 7 and 8/
- lack of sharpness of the variation limits of the admissible alterations of the helicopter parameters caused in this case by compressibility and stall, permits to extend the range of parameter alterations, but simultaneously it hampers both the synonymous programming of limits and the estimate of the helicopter quality.

For the evaluation of the degree of effect which the alteration of parameters exerts on the properties, the process of calculating the sensibility factors $\Delta W_j/\Delta p_i$ was programmed. Those factors are calculated by means of estimating the alteration of the group of deduced qualities in result of the alteration of one parameter by a unit. The conventionally designated here unit of parameter increment, may be established in an absolute magnitude, in percentage to the magnitude of the parameter, and in a proper proportion with regard to other parameters, or else recognizing the limits of the design variation of the parameter. The sensibility factors permit also to estimate the couplings of the parameters and qualities in order to decompose the multiobjective optimization into particular problems. Coupling, for instance of qualities "j" and "j+1" by the parameter "i" is carried out by estimating the value of factors $\triangle W_j/\triangle p_i$ and $\triangle W_{j+1}/\triangle p_i$.

The variation of the group of properties caused by a consecutive alteration of selected parameters by a unit is shown in fig. 9.

Auxiliary analyses, as well as the deduced sensitivity factors, enhance the knowledge of physical dependence of the actual course of relations, and develop the intuition of the person in charge of the optimization process. Therefore, the possibility of estimating the variation of the group of helicopter properties, due to a simultaneous alteration of the group of helicopter parameters, was programmed in column D. The sensitivity factors indicate in which direction a single parameter of the helicopter should be altered, while the effect of the variation of a selected group of parameters is estimated in column D. The length of the step of parameter variations is arbitrarily estimated by the person in charge of the process. Nevertheless, after several steps, it is generally possible to narrow in a large degree without any significant difficulties, the range of admissible solutions, keeping within even narrower limits of properties $W_{j\ min} < W_j < W_{j\ max}$, than the requirements laid down by the regulations for helicopter construction.

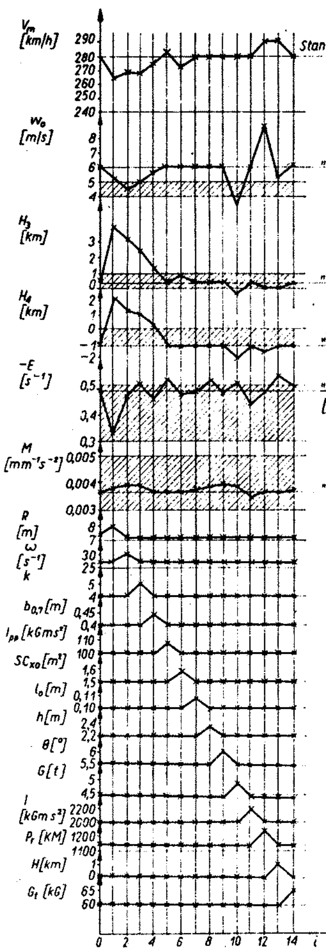

Fig.9. Auxiliary chart for estimating the sensibility factors

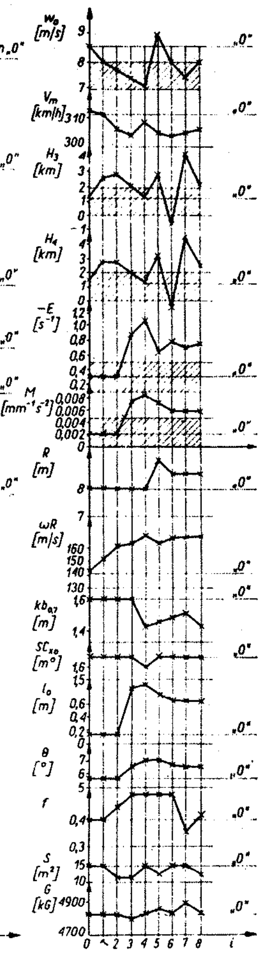

Fig.10. Exemplary illustration of the procedure of the optimization process /time-history/ Ranges of admissible properties /criteria/ are limited by shaded areas

Column D is characterized by a high degree of interaction between the designer and the analysis program.

An exemplary histogram of narrowing the range of solutions is given in fig.10.

The growing tendency to introduce, as far as possible, the automation of calculation processes has been reflected by the organization of program in column E. In this case, the technique of finding the extreme of one estimate for a multivariable group of the helicopter parameters, has been programmed. The techniques of numerical searching for the extreme may be various. The group of techniques, assembled in the AESOP system is described in [1]. One of the simpler techniques was used, namely, the consecutive penetration of the nearest environment of the initial group of parameters by the preset magnitudes $\Delta p_i$, and the selection of a larger function of the objectiv.

Difficulties were generally encountered, resulting mainly from the necessity of displacement along the limitation. It was therefore necessary to search the areas of the environment by deducing a bivariant chart according to the method programmed in column B. Decomposition into fragmentary problems enables an independent single - objectiv optimization of the selected parameter set in the aforementioned manner. This concerns such elements of the construction as the tail rotor, or the control system elements of the helicopter. Qualities, coupled to each other /as for instance the hovering flight ceiling, level air speeds, maximum rate-of-climb with one engine operative/ require the introduction of a single-dimensioned scale of the estimates for the maximilizing of the sum of those estimates.

**Examples of estimate distribution for the ceiling, maximum air speed and flight qualities are given in fig. 11.**

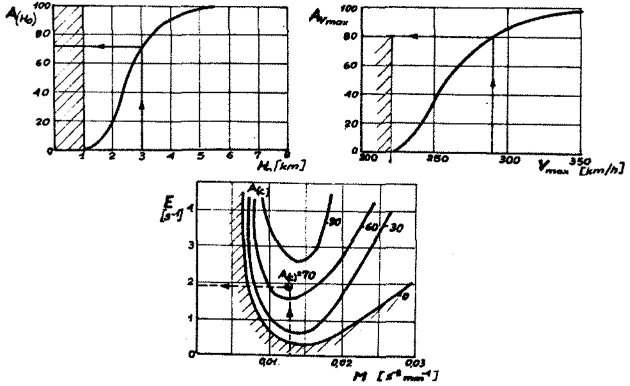

**Fig.11. Exemplary scales of estimates for the ceiling, maximum air speed, and flight qualities**

**The arbitration pertains here to the introduced function of estimates. One can have reservations as to their procedure outlined in accordance with subjective feeling, but nevertheless the designed estimates solely the satisfactory result, and accepts it as a better or worse solution, regardless of the means by which it was obtained.**

Although the automation of the calculation procedure is very interesting, and will become more so in the future, the necessity of constant interference into the calculation process, labor consumption, and the difficulty of programming limitations of all types, as well as the parameter variation limits /which also are often coupled to each other/, tends to limit the applicability of this method to a few steps.

The general estimate of the helicopter quality is still subjective, and belongs to the person in charge of the optimization, basing on the deduced group of helicopter qualities, so that it is easier to handle the procedure of column D.

## 3. OPTIMIZATION AT THE STAGE OF THE TECHNICAL PROJECT

After establishing in the first approximation the main parameters of the configuration rotor - wings /$\omega R$, $R$, $kb_{0.7}$, $S$, $c_{0.7}$/ during the previous stage, a detailed optimization of the remaining parameters, such as chord and aerofoil distribution along the span of the wing and rotor blade, is being carried out at this stage.

This task requires utilization of an accurate mathematical model of the helicopter.

This is connected with the volume of the program, time consumption by the calculations, and the necessity of adequate verification of the calculation procedures.

It has also an effect on the structure of the accepted diagram of optimization, shown in fig. 12.

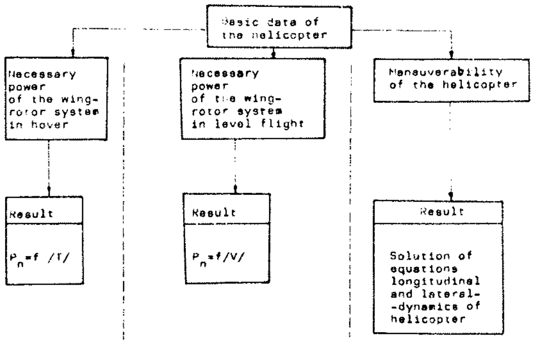

Fig.12. Diagram of the optimization process at the stage of the technical design of the helicopter

The considerable volume of the calculation programs permits to include only one estimate in a single program. With reference to this, the optimization system was reduced to a group of three independent estimates which were the equivalent of performance and flight qualities of the previous stage.

The block for estimating the qualities in a vertical flight is reduced to an estimate of the necessary power absorbed by the rotor-wing system in hovering flight. The block estimating the qualities in level flight, estimates the necessary power for the air speed selected in the first stage. The block estimating the manoeuvrability of the helicopter utilizes the model of numerical simulation for the preset manoeuvre or for calculating the aerodynamical derivatives of the system in the motion of pitching and rolling. The mathematical model of the rotor-wing configuration, in order to estimate the distribution of its geometrical parameters along the span of the blade and wing, must utilize the theory of the blade element. For the estimate of interference, only the vortex theory of the system may be of assistance. In the first approximation, the simplified geometry of wakes may be employed. If it is possible to utilize a high class computer, it is advisable to take into account the deformation of the wake. It must be stressed that the exact model of the system serves also to verify the simplified model in the first stage, and provides as well the correction factors for estimating the mean angles of the sweep of the flow around the helicopter elements.

In the interference model for estimating the hovering flight /fig. 13/, when the wing is flown around by a sub-rotor stream, the surface of discontinuity of the wing flow-around with stall /Kirchoff--Helmholtz model/, was replaced by a discrete vortex system. This permitted to calculate the magnitude of disturbance in the plane of the rotor by means of a method consisting in summing up the velocity induced by discrete vortex elements, in accordance with the Biot and Savart rule.

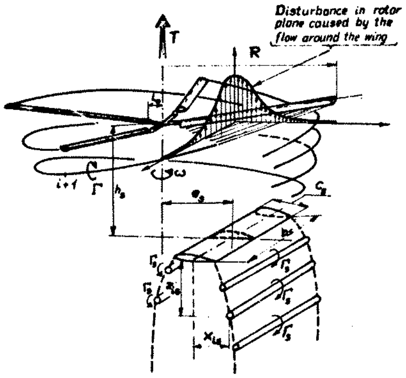

Fig.13. Model of the rotor-wing configuration for estimate of qualities in hovering flight

In level flight, for the model configuration shown in fig. 14, the interaction of the after-wing and after-rotor wake was solved similarly as in the case of hovering flight, and in the iteration process for the preset external loads, resulting from the calculations of the helicopter equilibrium in flight the subject problem was also solved.

The results of the solutions of the level flight are shown in fig. 15.

The helicopter dynamics in the pitching and rolling directions is solved similarly as in the first stage, only the equation coefficients are calculated more accurately by means of the numerical method. The directional manoeuvring, estimated in the previous stage in accordance with [3], is replaced by the model of simulation of the solution step by step of

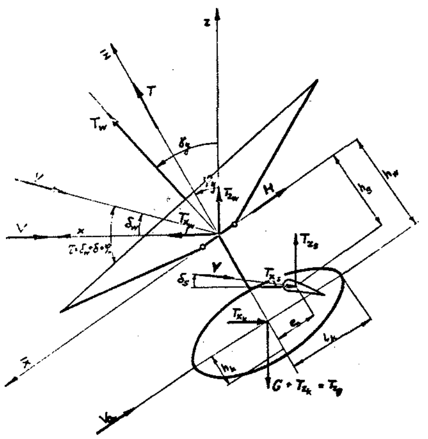

Fig.14. Model of the rotor-wing configuration for estimate of qualities in level flight

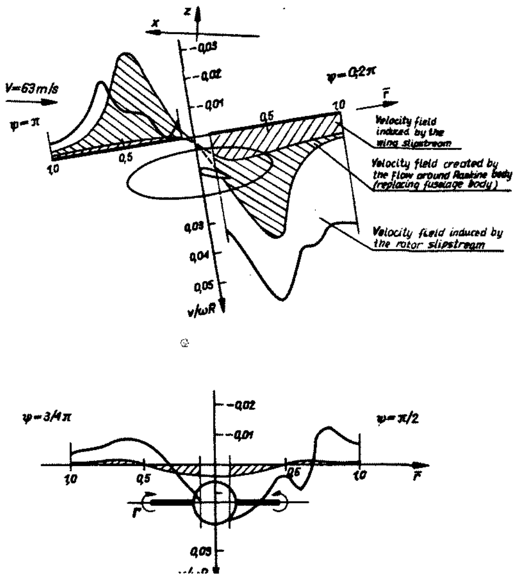

Fig.15. Results of calculations of the disturbance in the flow around the rotor in level flight

the helicopter turn around the vertical axis. The result of this solution is the distribution of loads and parameters of the tail rotor motion for the preset course of the control pulse /fig. 16/.

No automatic selection of parameters was introduced at this stage owing to the difficulties which are connected with employing a high-class computer.

Should a computer of such class be available, the algorithm of the automatic selection of parameters may be identical to the one in column E of the first stage.

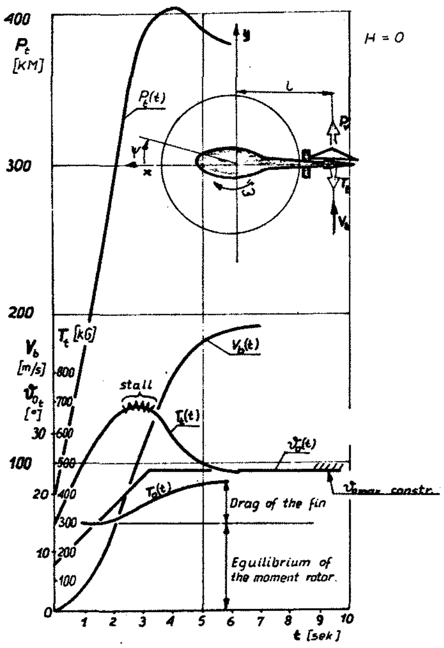

Fig. 16. Parameters of the helicopter motion, and of the tail rotor loads during a controlled turn around the vertical axis

## 4. CONCLUSION

Only some aspects of the optimization of a helicopter, equipped with an auxiliary wing, were discussed in this paper. It was not possible to deal with the problem of the mathematical model of a helicopter with regard to flying in an arbitrary manoeuvre, and in turbulent weather conditions - while this is considered the most outstanding disadvantage of the presented method.

Solely, an accurate model of a not stationary flight simulation would be the most suitable for an additional test of the helicopter behavior, particularly in such flight conditions as putting into autorotation, pulling-up, and turns. An estimate evaluation of those flight conditions may prove to be insufficient. Typical conclusions, which might be drawn from the aforementioned process, are as follows:

A wing may constitute the only possible solution, but such a circumstance is

**limited to only some specific configurations of the helicopter.**

**The profitability of the wing grows along with the increasing of requirements pertaining to air speed /fig. 8/. For articulated rotors, the wing cannot unload the rotor to a considerable extent since the flight qualities tend to drop in a significant degree, /fig. 17/. An additional increase of power in hovering** flight /fig. 7/ - due to the increase of the construction weight, caused by adding of the wing, may be compensated by a proper selection of the rotor, reduction of the r.p.m. rotor solidity, and increase of the radius, /fig. 18/.

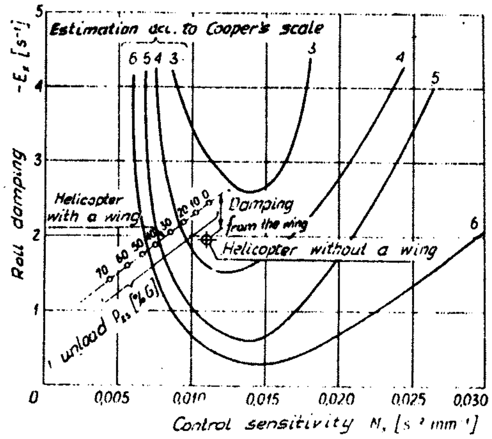

Fig.17. Estimate of the effect of relieving the rotor by the additional wing on the flight qualities on the direction of banking of a single-rotor helicopter with a weight of Q = 3750 kG, and at the air speed V = 200 km/h

It is a complex problem to draw conclusions of a general character.

Each particular case should be treated individually.

The introduction of new design developments /adjustment of the rotor r.p.m., a high-duty power plant, blade aerofoil/ may, in a certain range of velocity, eliminate the need for applying a wing.

However, in case of significant requirements pertaining to air speed, the **wing may be an effective means to solve many** controversial design requirements. **A skilfully performed optimization may reduce considerably the unfavorable effect of the wing.**

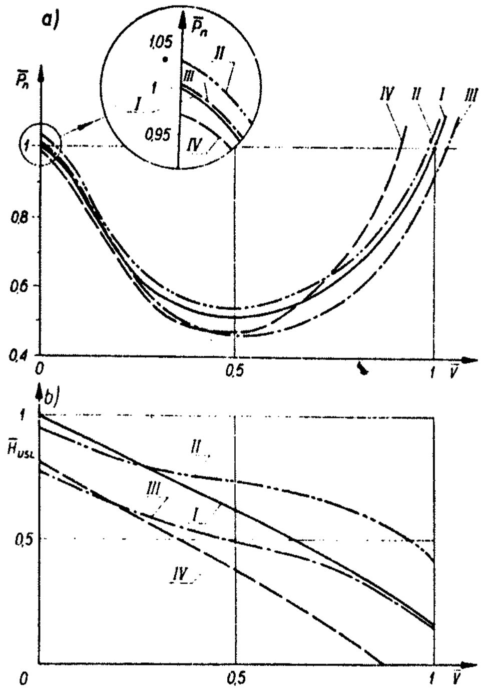

Fig.18. Exemplary illustrations: a/ relative balance of indispensable power, and b/ limits of separation for a classic rotor, and for helicopters equipped with a wing

I Classical helicopter $G = G_0$

II Classical helicopter, with wing added, without alteration of parameters of the rotor, $G = 1.012\ G_0$

III Helicopter with wing /version II/; rotor r.p.m. reduced by 10% $G = 1.02\ G_0$

IV Variant III, after removal of wing $G = 1.008\ G_0$

## SUPPLEMENT

### LIST OF DESIGNATIONS

| | | |
|---|---|---|
| R | m | - radius |
| $\omega$ | $s^{-1}$ | - angular velocity of the rotor |
| k | | - number of blades |
| $b_{0.7}$ | m | - blade chord at the distance of 0.7 R |
| $I_{pp}$ | $kGms^2$ | - inertia moment of the blade with reference to the horizontal hinge |
| $SCx_0$ | $m^2$ | - parasitic drag of the helicopter for zero pitching /inclination/ of the fuselage |

| | | |
|---|---|---|
| $l_o$ | m | distance of the horizontal hinge from the rotor axis of rotation |
| h | m | height of the hub above the center of gravity of the helicopter |
| $l_1$ | mm | maximum amplitude of longitudinal displacements of the control stick |
| $\theta$ | o | maximum amplitude of longitudinal displacements of the swashplate |
| f | | relief factor for relieving the rotor by the wing |
| S | $m^2$ | wing area |
| G | kG | weight of the helicopter in flight |
| I | $kGms^2$ | moment of inertia of the helicopter with reference to axis "y" /pitching/ |
| $P_r$ | KM | power of the engine |
| H | km | analysed altitude of flight of the helicopter |
| $l_s$ | m | wing span |
| $G_b$ | | weight of a single blade of the helicopter |
| $w_o$ | m/s | rate-of-climb /vertical flight/ without ground effect |
| $V_m$ | km/h | maximum air speed in level flight with a pre-set engine power |
| $H_1$ | km | critical height of the upper stall limit in hovering flight |
| $H_2$ | km | critical height of the lover stall limit in hovering flight |
| $H_3$ | km | critical height of the upper stall limit at an air speed of $V = V_m$ |
| $H_4$ | km | critical height of the lover stall limit at an air speed of $V = V_m$ |
| E | $s^{-1}$ | damping on the direction of pitching of the helicopter in flight for $V = V_m$ |
| M | $s^{-2}mm^{-1}$ | sensitivity of controlling with the control stick displaced by 1 mm for the direction of pitching of the helicopter $V = V_m$ |

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